Final Review: Part 3

Wednesday, June 7, 2023 8:50 AM

problem: consider $f(x,y) = x^2y^2 - x^2 - y^2$. classify (2nd derivatives: min, max, saddle, ?) all critical points (where $\partial_x f$, $\partial_y f$ vanish) of f

a) compute $\partial_x f$ & $\partial_y f$

$\partial_x f = 2xy^2 - 2x$

$\partial_y f = 2x^2y - 2y$

b) find critical points: (x,y) such that $\partial_x f$ & $\partial_y f = 0$

$2xy^2 - 2x = 0$
$xy^2 - x = 0$
$x(y^2 - 1) = 0$
$x = 0$ or $y = \pm 1$

$2x^2y - 2y = 0$
$x^2y - y = 0$
$y(x^2 - 1) = 0$
$y = 0$ or $x = \pm 1$

* could also plug values from 1st equation into and to solve / want both equations to be zero @ same time *

- $x = 0$ & $y = 0$
- $y = 1$ & $x = 1$
- $y = 1$ & $x = -1$
- $y = -1$ & $x = 1$
- $y = -1$ & $x = -1$

5 critical points: $(0,0)$, $(1,1)$, $(1,-1)$, $(-1,1)$, $(-1,-1)$

c) compute $\partial_{xy} f$, $\partial_{yx} f$, $\partial_{xx} f$, $\partial_{yy} f$

$\partial_{xx} f = \partial_x(2xy^2 - 2x) = 2y^2 - 2 \rightarrow a$

$\partial_{yx} f = \partial_y(2xy^2 - 2x) = 4xy \checkmark \rightarrow b$

$\partial_{xy} f = \partial_x(2x^2y - 2y) = 4xy \checkmark \rightarrow c$ (match)

$\partial_{yy} f = \partial_y(2x^2y - 2y) = 2x^2 - 2 \rightarrow d$

a, b, c, d different for each critical

- = both -

+ = same sign roots

(x,y)	a	b	c	d	ad - bc	char. poly	type
$(0,0)$	-2	0	0	-2	4	$\lambda^2 + 4\lambda + 4$	max
$(1,1)$	0	4	4	0	-16	$\lambda^2 - 16$	saddle
$(1,-1)$	0	-4	-4	0	-16	$\lambda^2 - 16$	saddle
$(-1,1)$	0	-4	-4	0	-16	$\lambda^2 - 16$	saddle
$(-1,-1)$	0	4	4	0	-16	$\lambda^2 - 16$	saddle

* $\lambda^2 - \lambda(a+d) + (ad-bc)$ *

* $ad - bc = +$, $a = -$ → both $-$ = max

$ad - bc = +$, $a = +$ → both $+$ = min

$ad - bc = -$ → one $+$, one $-$ = saddle *

skills for final to also study / work on:

- distances: point to line (cross) & point to plane (dot)
- trajectories: $r(t) = \langle 3t^2, \cos t, e^{-t} + t^2 \rangle$

  $v(t) = r'(t) \longrightarrow$ speed $= |v(t)|$

  $a(t) = v'(t) = r''(t)$

- intersections & spheres (ball) ↑ filled in sphere

* could ask about trajectory & intersection / sphere:
@ time t, where around sphere (inside, outside, on)? *